NS – 312

V Semester B.A./B.Sc. Examination, November/December 2016
(Semester Scheme) (Repeaters – Prior to 2016-17)
(NS – 2013-14 and Onwards)
MATHEMATICS – VI

Time : 3 Hours

Max. Marks : 100

***Instruction** : Answer **all** questions.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) Solve $(yz + xyz)dx + (zx + xyz)\,dy + (xy + xyz)\,dz = 0$.

2) Verify the condition for integrability $2yzdx + zxdy - xy\,(1 + z)dz = 0$.

3) Form a partial differential equation by eliminating the arbitrary constants $x^2 + y^2 + (z - c)^2 = a^2$.

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4) Solve $p = e^q$.

5) Solve Lagrange's linear equation $xp + yq = z$.

6) Solve $(D^2 + 4DD' - 5D'^2)z = 0$.

7) Using Rodrigue's formula, obtain expression for $P_0(x)$ and $P_1(x)$.

8) Show that $P_n(-x) = (-1)^n P_n(x)$ using generating function.

9) Starting from the expressions of $J_{1/2}(x)$ and $J_{-1/2}(x)$ in the standard form, prove that $\int_0^{\pi/2} \sqrt{x}\, J_{1/2}(2x)\, dx = \frac{1}{\sqrt{\pi}}$.

10) Using the expansion of $e^{\frac{x}{2}\left(t - \frac{1}{t}\right)}$, show that $J_n(-x) = (-1)^n J_n(x)$.

11) Using the recurrence relation $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$, show that $J_{-3/2}(x) = -\sqrt{\frac{2}{\pi x}}\left[\frac{x \sin x + \cos x}{x}\right]$.

12) Prove that $\nabla E = E\nabla$.

13) Construct Newton's divided difference table for the following

x	1	3	6	11
f(x)	4	32	224	1344

14) Express $3x^3 + x^2 + x + 1$ in a factorial notation.

15) Evaluate $\Delta^{10}\left[(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)\right]$ with $h = 1$.

16) Write the trapezoidal rule for finding $\int_a^b f(x)\,dx$.

17) Explain (i) Deterministic and (ii) Stochastic mathematical models.

18) In a population of birds, the proportionate birth rate and death rate are both constant, being 0.45 per year and 0.65 per year respectively. Formulate a model of the population and solve.

19) In the case of modelling of a projectile motion without air resistance, find the maximum range on the horizontal.

20) What are the assumptions to be made in getting partial differential equation model for a vibrating string ?

II. Answer **any four** questions : **(4x5=20)**

1) Verify integrability condition and hence solve
$(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0$

2) Solve $\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$.

3) Find the complete integral of $z^2(p^2 + q^2 + 1) = 1$.

4) Solve by Charpit's method $px + qy = pq$.

5) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$.

6) An insulated rod of length l has its ends A and B maintained at 0°C and 100° C respectively until steady state conditions prevail. If B is suddenly reduced to 0°C and maintained at 0° C, find the temperature at a distance x from A at time t.

OR

III. Answer **any three** questions : **(3×5=15)**

1) Prove that $(n+1)P_{n+1}(x) = (2n+1)\, x\, P_n(x) - n\, P_{n-1}(x)$.

2) Expand the function $f(x)=\begin{cases}0 & \text{in } -1<x<0\\ x & \text{in } 0<x<1\end{cases}$ interms of Legendre polynomials.

3) Show that $P_n(x) = P'_{n+1}(x) - 2xP'_n(x) + P'_{n-1}(x)$.

4) Prove that $\frac{d}{dx}\left[x^{-n}J_n(x)\right] = -x^{-n}J_{n+1}(x)$.

5) Prove that $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{(3-x^2)\sin x}{x^2} - \frac{3}{x}\cos x\right]$.

IV. Answer **any four** questions : **(4×5=20)**

1) Obtain the function whose first difference is $2x^3 + 5x^2 - 6x + 13$.

2) By separation of symbols prove that

$$u_x = u_{x-1} + \Delta u_{x-2} + \ldots + \Delta^{n-1}u_{x-n} + \Delta^n u_{x-n}$$

3) Evaluate f(7.5) from the table

x	1	2	3	4	5	6	7	8
f(x)	1	8	27	64	125	216	343	512

using Newton's backward interpolation formula.

4) Using Lagrange's interpolation formula find f(10) from the data.

x	5	6	9	11
f(x)	12	13	14	16

5) The population of a certain town is given below, find the rate of growth of population in 1971.

x : year	1931	1941	1951	1961	1971
y : population in thousands	40.62	60.80	79.95	103.56	132.65

6) Using Simpson's $\frac{1}{3}^{rd}$ rule, evaluate $\int_0^6 \frac{dx}{1+x^2}$ taking 6 equal parts.

V. Answer **any three** questions : (3×5=15)

1) Explain population growth model to show that $x(t) = x_0e^{at}$ and discuss the cases (i) $a>0$ (ii) $a<0$ and (iii) $a = 0$.

2) A breeder reactor converts respectively stable uranium – 238 into the isotope plutonium – 239. After 15 years, it is determined that 0.043% of the initial amount y_0 of the plutonium has disintegrated. Find the half-life of this isotope if the rate of disintegration is proportional to the amount remaining.

3) Form the differential equation of the free damped motion in the case of mass-spring Dashpot and discuss (i) over damped and (ii) critically damped cases.

4) Describe the projectile motion under gravity with air resistance and formulate the mathematical model.

5) Find the current I(t) in an RLC – circuit with R = 100 ohms, L = 0.1 henry, $C = 10^{-3}$ farad, which is connected to a source of voltage E(t) = 155 sin (377t).